and if conditions of Eq. (11) cannot be met then,

$$\Delta r(\epsilon + \alpha_T) < K_{cr} \tag{12}$$

where α_T is some trim angle caused by external configurational asymmetry. At high altitude prior to the onset of significant ablation, this term is probably negligible. For negative spin directions, the inequality signs in Eq. (11) are reversed.

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Application of Transformation Methods to Wedge-Shock/Boundary-Layer Interactions

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THE need for a method for the prediction of changes in the properties of the compressible turbulent boundary layer in the region of interaction with wedge or corner-induced oblique shock waves led to the development of the method presented here. The successful application of compressible boundary-layer transformation techniques to related problems by many authors, notably the application of boundary-layer transformation techniques to incident-reflecting-shock-wave/boundary-layer interaction problems by Seebaugh, Paynter, and Childs,[1] suggested the application of this technique to the wedge-shock/boundary-layer interaction problem.

General Properties of the Power-Law Transformation

The basic relationships are derived following the analysis of Ref. 1. The physical coordinates are transformed to a new set of coordinates for a corresponding incompressible flow

$$X(x) = x \tag{1}$$

$$Y(x,y) = \int_0^y \rho/\rho_e \, dy \tag{2}$$

It is assumed that the stream function, ψ, is invariant under the transformation and that the physical compressible boundary layer velocity profile may be expressed as a typical turbulent power-law profile, i.e.,

$$u/u_e = (y/\delta)^{1/n} \tag{3}$$

A similar relationship is assumed for the transformed flow

$$U/U_e = (Y/\Delta)^{1/N} \tag{4}$$

The assumption of constant energy in the boundary layer requires that the local stagnation enthalpy in the boundary layer be equal to the freestream stagnation enthalpy. For a perfect gas, this results in a simple relationship between local velocity and static temperature. The maximum attainable velocity, u_{max}, is that velocity corresponding to zero static temperature. If ϕ is defined as the ratio of u_e to u_{max}, this relationship becomes

$$T = [1/\phi^2 - (u/u_e)^2]u_e^2/2C_p \tag{5}$$

Through the use of the equation of state for a perfect gas and the assumption that the static pressure is constant through the boundary-layer thickness,

$$\delta = \int_0^\Delta T/T_e dY \tag{6}$$

Substitution of Eq. (5) into Eq. (6) and evaluation of the integral yields

$$\delta = D\Delta \tag{7}$$

where

$$D = (1 - \phi^2)^{-1}[1 - N\phi^2/(2 + N)] \tag{8}$$

From the relation between the physical and the transformed velocity and the definition of the stream function,

$$U = (\partial\Psi/\partial Y) = (\rho_e/\rho_0)u = (\rho_e/\rho)(\partial\psi/\partial y) \tag{9}$$

The invariance of the stream function requires that

$$\Psi(Y = \Delta) = \psi(y = \delta) \tag{10}$$

From Eqs. (9) and (10),

$$\int_0^\Delta \rho_0 U dY = \int_0^\delta \rho u dy \tag{11}$$

Evaluation of the integral on the left-hand side of Eq. (11) and further simplification yields

$$\frac{N/(1 + N)}{[1 - N\phi^2/(2 + N)]} = \int_0^1 \frac{(y/\delta)^{1/n}\, d\left(\frac{y}{\delta}\right)}{[1 - \phi^2(y/\delta)^{2/n}]} d\left(\frac{y}{\delta}\right) \tag{12}$$

Evaluation of the integral in Eq. (12) for various Mach numbers and compressible power-law exponents, $1/n$, yields the results shown in Fig. 1.

Wedge-Shock/Boundary-Layer Interaction

The control volume selected along with other necessary geometric quantities for the two-dimensional wedge-shock/boundary-layer interaction model is shown in Figure 2. The streamwise boundaries of the control volume are the compression surface downstream of the corner and a streamline passing through the corner shock in the inviscid region of the flow. The flow in the boundary layer passing through the control surfaces is assumed to be parallel to the compression surfaces. This precludes the existence of large flow disturbances in the boundary layer upstream of the compression corner; this is substantiated by experimental data in Ref. 2 where it was found that, in the absence of separation, the upstream influence was considerably less in extent than a length equivalent to one boundary-layer thickness. This requirement and the assumption of negligible wall shear stresses on the compression surface downstream of the compression corner within the interaction control volume are the most significant simplifications in the formulation of the model.

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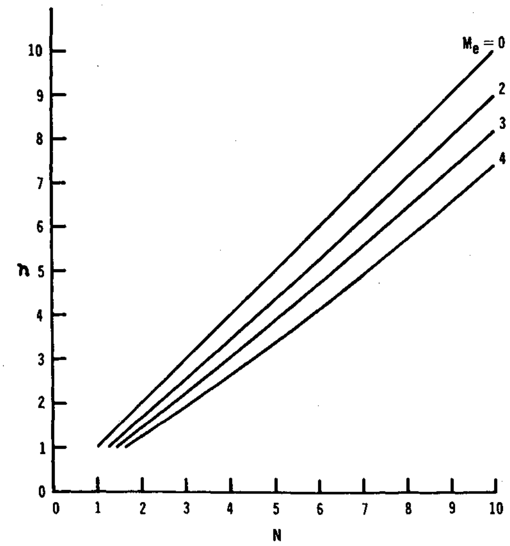

Fig. 1 Relation between n and N for boundary-layer transformation.

External flow properties are determined solely from the two-dimensional inviscid corner-shock solution as presented, for example, in Ref. 3. The pressures as determined from the inviscid solution are, as a result of the previous assumptions, uniform over the control volume boundaries in regions 1 and 2 (Fig. 2) and are equivalent to their respective freestream values.

The equation of conservation of mass is applied only to the mass flow in the boundary layer entering the control volume. This requires no entrainment of mass from the external flow into the boundary layer in the interaction region. This condition is consistent with the assumption of negligible wall shear stresses in the interaction region and is, also, one of the conditions imposed in the analysis of incident-reflecting-shock/turbulent-boundary-layer interactions in Ref. 1. The requirement of conservation of mass within the boundary layer is expressed as

$$\int_0^{\delta_1} \rho_1 u_1 dy' = \int_0^{\delta_2} \rho_2 u_2 dy \tag{13}$$

After nondimensionalization of variables, evaluation of the integrals, and simplification, Eq. (13) becomes

$$\frac{\Delta_2}{\Delta_1} = \frac{P_1}{P_2}\frac{Me_1}{Me_2}\left(\frac{Te_2}{Te_1}\right)^{1/2}\frac{(1+N_2)}{N_2}\frac{N_1}{(1+N_1)} \tag{14}$$

The equation of conservation of momentum is applied to the control volume as shown in Fig. 2. The control volume selected extends out from the compression surface sufficiently

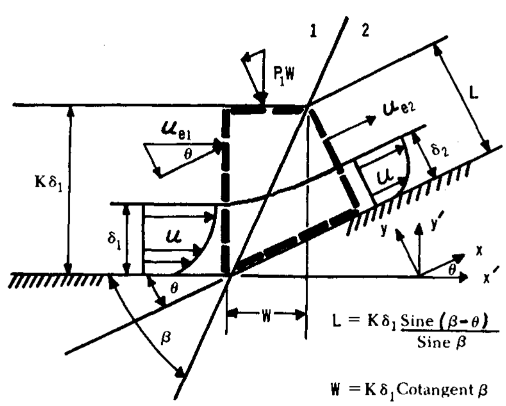

Fig. 2 Two-dimensional wedge-shock/boundary-layer interaction model.

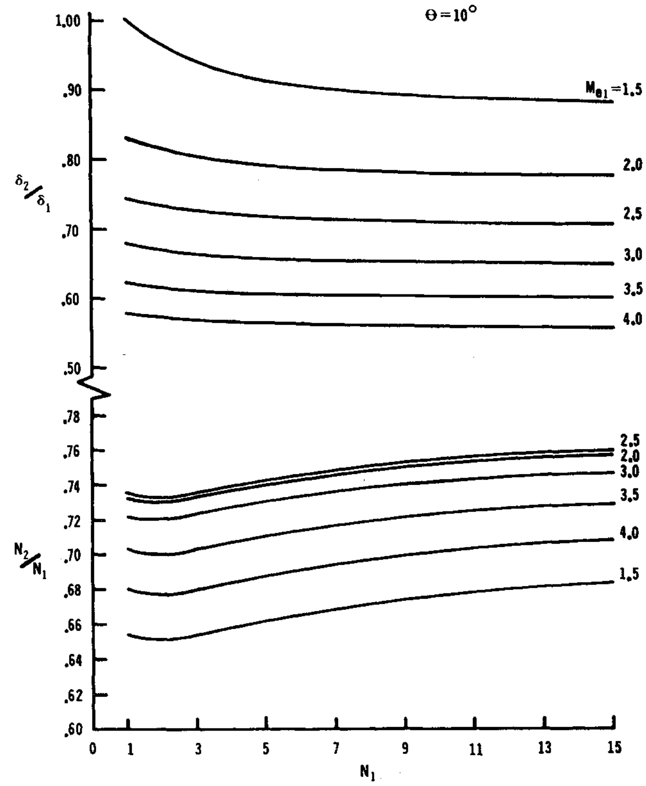

Fig. 3 Theoretical results for 10° turning.

far to enable the pressure on the outer boundaries to be specified from freestream conditions. Specification of corresponding pressures at the outer edge of the boundary layer (as in the analysis of Ref. 1) is not possible in this case since no intermediate region of inviscid flow is present. Wall shear stresses are neglected in the momentum balance under the assumption that the interaction takes place over a small physical length. This does not imply that wall shear stresses can be neglected on the compression surfaces upstream or downstream of the interaction region, and any subsequent calculation of boundary layer development downstream of the interaction must include the appropriate wall shear stress.

The requirement of conservation of momentum for the control volume as shown in Fig. 2 is

$$P_1K\delta_1\cos\theta - P_1K\delta_1\sin\theta\cot\beta - P_2K\delta_1\sin(\beta-\theta)\times \csc\beta = \int_0^{\delta_2}\rho_2u_2^2dy + \int_{\delta_2}^{K\delta_1\frac{\sin(\beta-\theta)}{\sin\beta}}\rho_{e2}u_{e2}^2dy - \int_0^{\delta_1}\rho_1u_1^2\cos\theta dy' - \int_{\delta_1}^{K\delta_1}\rho_{e1}u_{e1}^2\cos\theta dy' \tag{15}$$

After nondimensionalization of variables, evaluation of the integrals, and simplification through the use of the inviscid oblique shock momentum relationship, Eq. (15) becomes

$$\frac{\Delta_2}{\Delta_1} = \frac{P_1}{P_2}\frac{M_{e1}^2}{Me_2^2}\cos\theta\left[\frac{D_1 - N_1/(2+N_1)}{D_2 - N_2/(2+N_2)}\right] \tag{16}$$

Final Solution

Equations (14) and (16) are two independent relations for Δ_2/Δ_1. By equating their right-hand sides, a relationship between N_1 and N_2 is obtained.

$$\left[D_2 - \frac{N_2}{2+N_2}\right]\frac{(1+N_2)}{N_2} = \frac{Me_1}{Me_2}\left(\frac{Te_1}{Te_2}\right)^{1/2}\times \left[D_1 - \frac{N_1}{2+N_1}\right]\frac{(1+N_1)}{N_1\sec\theta} \tag{17}$$

It can be shown from the definition of ϕ and Eq. (8) that

$$D - N/(2 + N) = 2T_0/[(2 + N)Te] \quad (18)$$

Through the use of the above, and the fact that the total temperature, T_0, is constant throughout the interaction, Eq. (17) becomes

$$(1 + N_2)/(N_2^2 + 2N_2) = \alpha \quad (19)$$

where

$$\alpha = \frac{(1 + N_1)}{(N_1^2 + 2N_1)} \frac{Me_1}{Me_2} \left(\frac{Te_2}{Te_1}\right)^{1/2} \cos\theta \quad (20)$$

Explicit solution of Eq. (19) for the quantity N_2 yields

$$N_2 = (1 + 0.25\alpha^{-2})^{1/2} + 0.50\alpha^{-1} - 1 \quad (21)$$

Equations (21) and (20) determine N_2 from known quantities in the region of interaction. The corresponding compressible power-law reciprocal exponent, n_2, may be determined from N_2 and M_{e2} through the use of Fig. 1. The boundary-layer thickness ratio δ_2/δ_1 is then obtained from Eq. (14) or (16) through the use of Eq. (7).

Analytical Results

Sample analytical results are presented in Fig. 3. The ratios of δ_2/δ_1 and N_2/N_1 are presented as a function of N_1 for a range of freestream Mach numbers (M_{e1}) and a compression corner angle of 10°. It can be seen from Fig. 3 that the predicted effect of the interaction is to increase the boundary layer velocity-profile exponent ($1/n$), which essentially "weakens" the boundary layer and increases its susceptibility to separation. The results indicate a net reduction in boundary layer thickness due to the increase in density of the fluid in the boundary layer as it is influenced by the compression field downstream of the shock wave. Although no experimental data are available to verify these analytical results for this type of shock/boundary-layer interaction, data are presented in Refs. 4 and 5 which clearly indicate similar boundary layer behavior in incident-reflecting-shock/boundary-layer interactions in cases where separation is not evident.

It should be emphasized that these theoretical results do not account for real viscous effects, and that boundary-layer separation is a significant or dominating factor in flow situations where it is present. The theoretical results of this paper should not be applied in cases where the limits of incipient boundary-layer separation have been reached. Experimental data on incipient separation for this kind of interaction were obtained by Kuehn[6] for boundary layer Reynolds numbers ($R_{e\delta}$) of from 1.5 to 7.5 × 10^4. Data for Reynolds numbers ($R_{e\delta}$) of from 1.5 to approximately 7 × 10^6 are presented in Ref. 2. An approximation for the range of Reynolds numbers considered for Mach numbers up to 4.0 is the following:

$$\theta \text{ incipient separation} \geq (5.5M_{e1})^\circ \quad (22)$$

The aforementioned references should be consulted if θ exceeds 5.5 Me_1.

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Curvature Effects in the Laminar and Turbulent Freejet Boundary

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Nomenclature

C = normalized curvature parameter
E = entrainment parameter
f = normalized stream function
k = curvature of the "major streamline"
k_0 = curvature parameter
p = pressure
R = radius of curvature of the major streamline
u = x component of velocity
U_1 = x component of velocity along major streamline
U_{max} = maximum x component of velocity
v = y component of velocity
x = coordinate along major streamline
y = coordinate normal to major streamline
ϵ = eddy viscosity
η = normalized y coordinate
κc = constant in the expression for eddy viscosity
μ = viscosity
ν = kinematic viscosity
ρ = density
σ = scaling factor in normalized y coordinate
τ = shearing stress
ψ = stream function

If a nearly uniform stream passes from a nozzle, say, into a stagnant region there is a mixing between the jet issuing from the nozzle and the stagnant fluid. Just downstream of the point where the jet enters the stagnant fluid the flowfield is described by the so-called "freejet boundary." Normally for such flows there is no pressure gradient across the jet and the streamlines have no curvature other than that associated with normal spreading of the jet. There are physical cases in which there is a pressure gradient across the flow and as a result the streamlines are curved. Perhaps the most timely example of such a curved freejet boundary occurs in a fluid amplifier device which employs the "Coanda" effect. In such a device the jet entrains fluid between the main body of the jet and a wall next to the jet, resulting in a reduction of the pressure on the side of the jet next to the wall. The reduced pressure causes a deflection of the jet toward the wall until an equilibrium situation results.

A related physical situation occurs in the jet flap[1] where a jet at one velocity issues from the trailing edge of an airfoil and mixes with parallel streams of different velocities. A curved jet results in which a pressure difference is maintained by the curved jet which involves not one but two freejet boundaries.

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